Power transfer characteristics among \( N \) parallel single-mode optical fibers

Ye Wang, Dajian Xue, Xuanhui Lu*

State Key Laboratory of Modern Optical Instrumentation, Department of Physics, Optics Institute, Zhejiang University, Hangzhou 310027, PR China

Received 22 August 2007; received in revised form 10 January 2008; accepted 11 February 2008

Abstract

Based on the coupled-mode theory, the power transfer among “- - -” arranged parallel single-mode optical fibers has been investigated. The analysis shows that the distances between each two of the \( N \) fibers centers have effects on the coupling coefficient and power transfer. The solution of the coupled equations for three parallel single-mode optical fibers is given, and is studied for different initial conditions comparatively. Numerical simulations show that power transfer will be periodical during coupling among parallel single-mode optical fibers. These results can be extended to multi-parallel single-mode optical fibers.

Keywords: Optical fiber array; Modes coupling; Power transfer

1. Introduction

It is a well-known fact that if the optical fiber cores are brought close enough to each other, wave coupling and power transfer among optical fibers happens due to the interaction of the extended evanescent optical fields right outside of the cores. The coupling is important to divide and mix optical signals for various optical fiber components, such as fiber couplers [1,2] and power dividers [3]. For example, the well-known phenomenon of evanescent wave coupling has wide applications such as design of waveguide switches [4], an important passive device in optical communications. However, in some other cases, an avoidance of wave coupling effects among “- - -” arranged optical fibers is needed. A representative example is pivotal components in a new scanning system for imaging.

*Corresponding author. Tel./fax: +86 571 87953232. E-mail address: xhlu@zju.edu.cn (X. Lu).

In previous studies, Snyder [5] presented a general coupled-mode theory and obtained conclusive results that could be used in a number of different wave propagation problems. In his study, the solution of the coupled equations for two parallel optical fibers was further presented. He also studied the coupling effects on the situation of a special fiber surrounded by other \( N \) identical fibers. However, it is only a special case among the generic two-body coupling problems. McIntyre [6] et al. extended Snyder’s results to the coupling of multi-mode fibers, and discussed the power transfer between a special fiber and six other non-identical fibers, which is also a generic two-mode coupling problem. Chang et al. [7] analyzed numerically the cases of power transfer between a signal-mode fiber and a multi-mode fiber. In general cases, two-mode coupling of many “- - -” arranged parallel optical fibers cannot be easily solved. To determine the details of the power transfer among the “- - -” arranged \( N \) fibers, it is required to solve the equations for \( N \) systems. In this paper, we derived the solution of the coupled equations of three parallel...
2. The coupled-mode method

We consider \( N \) infinitely long and parallel identical cores embedded in the cladding background, as shown in Fig. 1. The refractive index of the SM core and the cladding are denoted as \( n_1 \) and \( n_2 \), respectively. The radius of the SM core and the distance between two centers of the neighboring cores are shown as \( r \) and \( d \), respectively.

We adopt the coupled-mode equations derived in Ref. [5], in which only the closest coupling is taken into account. For the present system of \( N \) (\( N>3 \)) parallel fibers of no loss, the equations are as follows:

\[
\frac{dA_i(z)}{dz} + i\beta_i A_i(z) = -iA_{i+1}(z)C_{12} \quad (i=1,2,\ldots, N-1) 
\]

(1)

\[
\frac{dA_m(z)}{dz} + i\beta_m A_m(z) = -iA_{m+1}(z)C_{m+1} + iA_{m-1}(z)C_{m-1} \quad (m=2,3,\ldots, N-1) 
\]

(2)

\[
\frac{dA_N(z)}{dz} + i\beta_N A_N(z) = -iA_{N-1}(z)C_{N-1} 
\]

(3)

The \( z \)-axis in Cartesian coordinates is taken to be parallel to the fiber axes. In Eq. (1)–(3), the subscripts \( m \) and \( N \) refer to the \( m \)th fiber and \( N \)th fiber, respectively. \( \beta \) represents the propagation constant of mode, and \( A(z) \) represents the coefficient of mode on \( z = \) constant. The symbols \( C_{ab} \) is the coupling coefficient between the \( a \)th and \( b \)th fibers, and is defined as

\[
C_{ab} = \frac{\omega}{2} \int_{A_b} (n_1^2 - n_2^2) e_a \cdot e_b \, ds 
\]

(4)

where \( \omega \) is the wave frequency, \( A_b \) represents the cross section of the \( b \)th core, \( e \) is the electric field vector of mode. All modes are assumed to travel in the positive \( z \) direction. All the propagation constants of mode are equal because the \( N \) SM fibers are identical, and all coupling coefficients are equal as well according to Eq. (4). Thus all the propagation constants of mode and all coupling coefficients can be shown as \( \beta \) and \( C \), respectively. It has been shown that the power transfer between the forward and backward modes is very small and can be neglected for the case where the coupling coefficients are much smaller than the propagation constants of mode, that is, the case satisfying the weak-coupling condition [5]. In the following examples, \( C \) is found to be much smaller than the accompanying value \( \beta \). Thus, we only consider weak-coupling cases.

\( \beta \) and \( C \) were given [5,6] in the following forms by introducing three dimensionless parameters \( U \), \( V \) and \( W \):

\[
\beta = \left( \frac{\left( \frac{2\pi n_1}{\lambda} \right)^2}{\frac{U^2}{\rho^2}} \right)^{1/2} 
\]

(5)

\[
C = \frac{\sqrt{\delta \rho V^2} K_0(W(d/\rho))}{\rho V^2 K_1(W)} 
\]

(6)

where \( \lambda \) is the wavelength of light in vacuum, \( K_s \) are modified Hankel functions, and \( \delta \) is defined as

\[
\delta = 1 - \left( \frac{n_1}{n_2} \right)^2 
\]

(7)

A dimensionless frequency \( V \) is defined to be

\[
V = \frac{2\pi \rho n_1 \sqrt{\delta}}{\lambda} = U^2 + W^2 
\]

(8)

\( U \) is given approximately as

\[
U \cong 2.405e^{-(1-\delta/2)V}. 
\]

(9)

3. Results and discussion

3.1. Solutions for two different initial conditions and discussion

When \( N = 3 \), the solutions to Eqs. (1)–(3) are given as

\[
\begin{align*}
A_1(z) & = q_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-i\beta z} + q_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-\sqrt{5}C_{12}z} \\
A_2(z) & = q_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{-i\beta z} + q_3 \begin{pmatrix} 1 \\ -\sqrt{5} \end{pmatrix} e^{-\sqrt{5}C_{12}z} \\
A_3(z) & = q_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-i\beta z} + q_3 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-\sqrt{5}C_{12}z}
\end{align*}
\]

(10)

where \( q_1, q_2, q_3 \) are determined by initial conditions. Two cases of unit power coupling are considered.
3.1.1. Excitation of the 1st fiber with unit power \( A_1(0) = 0 \) with \( A_2(0) = 1, \cdots A_3(0) = 0 \ldots \)

By combining this condition with Eq. (10), we obtain
\[
q_1 = 1/2, q_2 = q_3 = 1/2.
\]

The power of each fiber can be shown as \( P_i(z) = |A_i(z)|^2 \) \( (i = 1, 2, 3) \). If \( \lambda = 1.31 \mu m \), the refractive index of the core is \( n_1 = 1.458 \), the refractive index of the cladding is \( n_2 = 1.455 \), and the radiuses of the cores are \( r = 4 \mu m \). The results of the power coupling among the three fibers are shown in Fig. 2 for \( d = 12, 20, \) and \( 50 \mu m \), respectively. In all three cases, the three fibers appear to undergo significant power coupling and variations, as indicated by the power vs. distance curves shown in the figures. It is noticed in Fig. 2(a) that at the distance of 4.48 mm, almost all the power of the 1st fiber has been transferred to that of the 3rd fiber. In the cases of Fig. 2(b) and (c), the power of the 1st fiber drops to zero at \( z = 56.5 \) mm and 482.8 m, respectively, and the power of the 3rd fiber reaches the peak of unit power simultaneously. At the beginning of the propagation, the distance at which the power of the 1st fiber drops to half of its initial value is found to be 1.63 mm for the case of \( d = 12 \mu m \). And as the coupling strength decreases for the case of \( d = 50 \mu m \), the distance increases monotonously to 175.5 m. Initially, the power of the 2nd fiber is observed to gain most of its power from the 1st fiber. The maximum power point of the 2nd fiber depends on the coupling strength and the spacing between the two fibers, and reaches half of the maximum power value of the 1st and the 3rd fibers in all three cases. The maximum power point of the 2nd fiber is 8.965 mm at \( d = 12 \mu m \). The cases of \( d = 20 \) and 50 \( \mu m \) show the same variation trend of power. All three fibers have the periodical power transfer along the propagation distance.

It can be seen by analyzing Fig. 2(a)–(c) that the coupling period increases as the space \( d \) increases. The cause is that the coupling coefficient goes down observably with the increasing space between the two neighboring fibers, and the coupling strength among optical fibers falls dramatically accordingly. Thus the outward transfer velocity of the power of the 1st fiber also decreases. So the wave propagation distance is longer as the initial power of the 1st power drops to zero. Fig. 3 does show the relationship between the coupling coefficient and the space between the two neighboring fibers. When \( d > 32 \mu m \) (correspondingly \( C = 0.999 \)), the coupling is so weak that it can be neglected in this case.

![Fig. 2. Power variations versus propagation distance of three optical fibers according to the initial condition 1. Three cases with \( N = 3, d = 12, 20, \) and \( 50 \mu m \) are shown. Curves 1, 2, 3 represent the power variations of optical fibers 1, 2, 3 respectively. The unit power is defined as the original value of power when \( Z = 0 \), and it is the same as below.](image-url)

3.1.2. Excitation of the 2nd fiber with unit power \( A_2(0) = 1 \) with \( A_1(0) = 0, A_3(0) = 0 \)

The power coupling of another case is presented in Fig. 4. Now the initial condition is \( A_2(0) = 1, A_1(0) = 0, \) and
Using Eq. (10), we have \( q_1 = 0, q_2 = \sqrt{2}/4, q_3 = -\sqrt{2}/4 \), for \( d = 20 \mu m \). It can be observed that the variations are similar to those shown in Fig. 2(b). However, the power transfer of the 1st and the 3rd fibers is synchronous and their transfer velocities are also same. The maximal power of the 1st and the 3rd fibers reaches half of the total power. The Figs. 4 and 2(b) show that the power transfer velocities of the 1st and the 3rd fibers in Section 2 are faster than those in Section 1, and the coupling period in Fig. 4 is shorter than that in Fig. 2(b).

### 3.2. Coupling analysis when \( N > 3 \)

If the initial condition is \( A_1(0) = 1 \) and zero for other fibers, the Eqs. (1)–(3) with \( N = 4 \) and 5 can be solved numerically. Figs. 5 and 6 show the power variations versus propagation distance with \( d = 12 \mu m \) in the 4-fiber and 5-fiber systems, respectively. The coupling of 4 or 5 fibers is complicated. The power of each fiber oscillates in the propagation period, and the peak does not vary monotonously. In Fig. 5, the power peak value of the 1st fiber is unit power initially. After coupling and oscillation, it drops and then increases, until it reaches its secondary peak finally. The variations of the power of the 2nd and the 3rd fibers are similar to each other, increasing initially and decreasing later. However, they are different from the 1st one, which has the same trend as the 4th fiber. The coupling velocity of the 2nd fiber is larger than that of the 3rd fiber, and the 1st power peak of the 2nd fiber is higher than that of the 3rd one, which is around 0.55. The 4th fiber has the slowest coupling velocity, while it has higher power peak than the 2nd.

![Fig. 3. Coupling coefficient as a function of the distance between two optical fiber centers, derived from Eq. (6).](image1)

![Fig. 4. Power variations versus propagation distance according to the initial conditions 2 when \( N = 3, d = 20 \mu m \). Now curves 1, 3 are overlapping.](image2)

![Fig. 5. Power variations versus propagation distance when \( N = 4, d = 20 \mu m \), with initial conditions \( A_1(0) = 1 \) and zero for others.](image3)
and the 3rd fibers. The power of the 4th fiber occupies almost 97 percent of total power when it is at the peak of its power, with the powers of the 2nd and the 3rd fibers simultaneously transferred out, except the 1st one. So this case is different from that of \( N = 3 \). In other words, the fringe \( N \)th fiber cannot get all the power when the power of middle fibers is zero.

By comparing Figs. 5 and 6, it is found that there is little difference between the power variation of \( N = 4 \) and that of \( N = 5 \). The coupling is strong between the 1st and last fibers no matter. The maximal power of the \( N \)th fiber exceeds 90 percent of the total power. As the number of fibers increases, every oscillating peak value of the fringe fibers in a coupling period decreases, and the wave propagation distance gets short from the initial to the zero-power point.

4. Conclusion

In this paper, we have calculated the power transfer among many “---” arranged parallel single-mode (SM) optical fibers based on a coupled-mode analysis. Different initial coupling conditions and different numbers of optical fibers are studied comparatively. Numerical simulations and theoretical curves show that the power transfer varies regularly among \( N \) “---” arranged parallel SM optical fibers. The coupling becomes stronger when the space between the two neighboring fibers is smaller. When using \( N \) “---” arranged parallel optical fibers, the power coupling can be controlled in different applications if the space between two neighboring fibers and the propagation distance are adjusted properly. On the other hand, after investigating these results, the coupling can be avoided efficiently in the scan imaging system, where it is necessary to keep uniformity for every spot.

Acknowledgments

This work is supported by National Science Foundation of China (Grant no. 10334050) and Scientific Project of Zhejiang Province (Grant no. 2005c21003).

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